

CALCULATION OF S_{11} PARAMETER OF COPLANAR STRIP AND INFLUENCE OF TECHNOLOGY PROCESS

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Summary The paper is focused on a calculation of the S_{11} parameter of the coplanar strips. The primary parameters calculations of the coplanar strips are described in the first part. The scattering parameters are defined. The calculation of the S_{11} parameter and the comparison with the measured values are showed in the last part of the work.

1. INTRODUCTION

A microwave technology meets with term planar strip. The strip is created on a substrate. We know two technologies for the planar strip manufacturing - additive (thin-film, thick-film) and subtractive (etching) technology [1]. The coplanar strip (fig. 1) is part of the planar strip group.

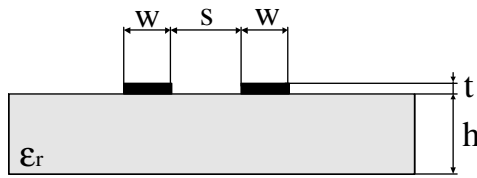


Fig. 1. Coplanar strip

An equivalent circuit (fig. 2) was used for the calculation of the coplanar strip. At the sometime the simplifying conditions was established. The transverse conductance was neglected. The strip is considered as homogenous and will be excited with harmonic current. The primary parameters calculation is first step for description of the strip.

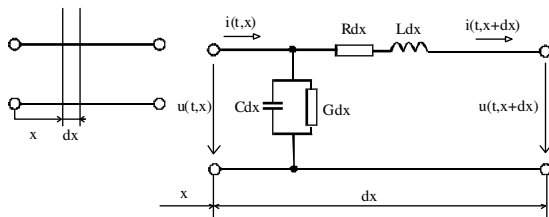


Fig. 2. Equivalent circuit

2. CALCULATION OF PRIMARY PARAMETERS

The equation (1) for specific resistance calculation can be used. The influence of an electric surface effect (skin effect) have to be included in the calculation, because of high frequencies are

supposed. According to literature [2] the path cross-section is reduced to ratio of a conductive path thickness divided by a skin depth (δ). The specific resistance is frequency variable, because the skin depth is frequency dependent. The final equation for the calculation of the specific resistance is at (2).

$$R = \rho \cdot \frac{l}{S} [\Omega] \quad (1)$$

ρ is resistivity, l is length of path and S is cross-section of path

$$R = \frac{\rho}{w \cdot \sqrt{\frac{2}{2\pi f \cdot \gamma \cdot \mu}}} [\Omega/m] \quad (2)$$

w is width of conductive path, f is frequency and μ is permeability of conductive path

The conformal transformation is used for the calculation of the specific capacity [1]. The advantage of the transformation conserves capacity. We do not backward transformation of the results. The influence of thickness substrate and conductive path is involved in this calculation. The final equation for capacity calculation is at (3).

$$C = \epsilon_0 \cdot \left[\epsilon_{ef} \cdot \frac{K(k')}{K(k)} + \left(2.3 \cdot 1.65^{-18 \frac{t}{w}} + 2 \right) \cdot \frac{t}{2w} \right] [F/m] \quad (3)$$

t is thickness of conductive path, ϵ_0 is permittivity of vacuum

$$\epsilon_{ef} = 1 + \frac{\epsilon_r - 1}{2} \cdot \frac{K(k') \cdot K(k_1)}{K(k) \cdot K(k'_1)}$$

$$K(k) = \int_0^1 \frac{dx}{\sqrt{(1-x^2) \cdot (1-k^2 x^2)}}$$

$$k = s/(s+2w)$$

$$k' = \sqrt{1-k^2}$$

$$k_1 = \frac{\sinh\left(\frac{\pi s}{4h}\right)}{\sinh\left(\frac{\pi(s+2w)}{4h}\right)}$$

$$k'_1 = \sqrt{1-k_1^2}$$

s is distance of conductive paths and h is thickness of substrate

The equation (4) for the calculation of the characteristic impedance is used for the calculation of the specific inductivity. So, the characteristic impedance must be calculated as first part. The equation (5) obtained from literature [1] was used for the calculation of the characteristic impedance. The equation for the calculation of the specific inductivity is at (6).

$$Z_0 = \frac{c}{\sqrt{\epsilon_{ef}}} \cdot L \quad (4)$$

$$Z_0 = \frac{1}{c \cdot \sqrt{C \cdot C_1}} \quad (5)$$

c is speed of light and C_1 is capacity of air dielectric

$$C_1 = \epsilon_0 \cdot \left[\frac{K(k')}{K(k)} + \left(2,3 \cdot 1,65^{-18 \cdot \frac{t}{w}} + 2 \right) \cdot \frac{t}{2w} \right]$$

$$L = \frac{\sqrt{\epsilon_{ef}}}{c} \cdot Z_0 [H / m] \quad (6)$$

3. CALCULATION OF S_{11} PARAMETER

The definitions of the scattering parameters are shown at figure 3. The characteristics of the two-port systems are not characterized by the applied voltages and currents, but by standardized incident voltage waves a_1 and a_2 and by reflected voltage waves b_1 and b_2 .

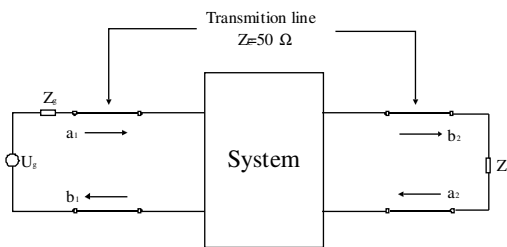


Fig. 3. Two-port system characterized by incident and reflected waves

These waves rise on input and on output transmission line which has characteristic impedance Z_C . Applied voltages and currents are related to voltage waves with equations (7). Incident and reflected waves are defined depending on applied voltages and currents by equations (8). The characteristic impedance Z_C can be generally complex number. But it is used as a real impedance, with a typical value $Z_C=50\Omega$. A reciprocal dependence of incident and reflected voltage waves are formulated by equations (9), which can be taken

in account as definition equations of scattering parameters of a two-port system [3], [4], [5].

$$\begin{aligned} u_1 &= (a_1 + b_1)\sqrt{Z_C} \\ u_2 &= (a_2 + b_2)\sqrt{Z_C} \\ i_1 &= (a_1 - b_1)/\sqrt{Z_C} \end{aligned} \quad (7)$$

$$\begin{aligned} i_2 &= (a_2 - b_2)/\sqrt{Z_C} \\ a_1 &= (u_1 + i_1 Z_C)/2\sqrt{Z_C} \\ a_2 &= (u_2 + i_2 Z_C)/2\sqrt{Z_C} \end{aligned} \quad (8)$$

$$\begin{aligned} b_1 &= (u_1 - i_1 Z_C)/2\sqrt{Z_C} \\ b_2 &= (u_2 - i_2 Z_C)/2\sqrt{Z_C} \\ b_1 &= s_{11}a_1 + s_{12}a_2 \\ b_2 &= s_{21}a_1 + s_{22}a_2 \end{aligned} \quad (9)$$

$$s_{11} = \left. \frac{b_1}{a_1} \right| a_2 = 0$$

$$s_{12} = \left. \frac{b_1}{a_2} \right| a_1 = 0$$

$$s_{21} = \left. \frac{b_2}{a_1} \right| a_2 = 0$$

$$s_{22} = \left. \frac{b_2}{a_2} \right| a_1 = 0$$

For the calculation of parameter S_{11} the mentioned definition (9) is used. With a substitution (8), the equation for the calculation of parameter S_{11} is at (10).

$$S_{11} = \left. \frac{b_1}{a_1} \right| a_2 = 0 \Rightarrow S_{11} = \frac{(u_1 - i_1 Z_C)/2\sqrt{Z_C}}{(u_1 + i_1 Z_C)/2\sqrt{Z_C}} = \frac{u_1 \left(1 - \frac{Z_C}{Z_{IN}} \right)}{u_1 \left(1 + \frac{Z_C}{Z_{IN}} \right)} = \frac{Z_{IN} - Z_C}{Z_{IN} + Z_C} \quad (10)$$

Where Z_{IN} is input impedance of two-port system, in this case it is the coplanar strip, which is ended with the characteristic impedance Z_C . The telegraphic equation is used for the calculation of input impedance. An idea of calculation procedure is evident from fig. 4. The final equation for input impedance is at (11).

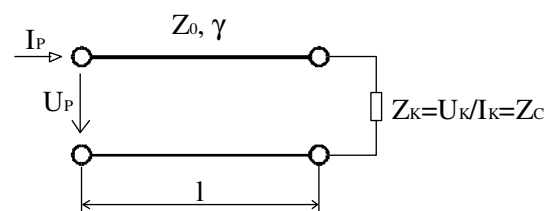


Fig. 4. Diagram for calculation of input impedance

$$Z_{in} = \frac{U_p}{I_p} = \frac{U(l)}{I(l)} = \frac{U_K \cosh(\gamma \cdot l) + Z_0 I_K \sinh(\gamma \cdot l)}{I_K \cosh(\gamma \cdot l) + \frac{U_K}{Z_0} \sinh(\gamma \cdot l)} \cdot \frac{Z_c}{\cosh(\gamma \cdot l)} \quad (11)$$

$$Z_{in} = \frac{U_K (Z_c + Z_0 \tanh(\gamma \cdot l))}{U_K \left(1 + \frac{Z_c}{Z_0} \tanh(\gamma \cdot l)\right)} = Z_0 \frac{Z_c + Z_0 \tanh(\gamma \cdot l)}{Z_0 + Z_c \tanh(\gamma \cdot l)}$$

4. TEST SAMPLES

Test samples were created for comparison of theoretic results. Design of test samples were limited by theoretic calculation. The equations used limit ratio width and distance of paths. A design had to match the requirement $1,25 \leq (s + 2w)/w \leq 10$. In that case the test samples have distance between conductive paths (s) 4 mm and width of conductive paths (w) 1 mm.

The samples were created with the subtractive technology, where the substrate FR4 was used as basic material. Because the part of designed samples contained conductive holes (vias), all samples were created with process for double-sided printed circuit boards with conductive holes [6]. The coplanar strip is formed by electrolytic copper and electroplated by tin, which serves as mask for etching. Designed samples are on fig. 5.

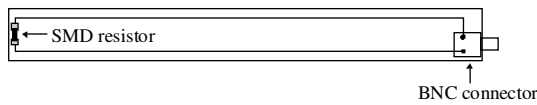


Fig. 5. Test samples

5. CALCULATED VALUES FOR DESIGNED COPLANAR STRIP

For the calculation of primary parameters and S_{11} parameter was used software Microsoft Excel from Microsoft Office XP. The software allows calculations with complex numbers as well as the software is able to create graphs. This software is suitable for the required calculation. The calculation was done for:

- distance between conductive paths 4 mm
- width of conductive paths 1 mm
- thickness of conductive paths 70 μm (basic copper thickness 35 μm and electrolytic layers of copper and tin)
- substrate thickness 1,5 mm
- relative permittivity of substrate 5
- frequency range from 0,5 GHz to 2 GHz

Results of theoretic calculation:

The specific resistance calculated according to equation (2) is for frequency 500 megahertz 5,88 Ω/m and for frequency 2 gigahertz 11,77 Ω/m .

The specific capacity calculated according to equation (3) is 21,79 pF/m. The rectangular method with step 0,001 was used for calculation of integrals.

The specific inductivity calculated according to equation (6) is 1,10 $\mu\text{H}/\text{m}$.

The S_{11} parameter calculated according to equation (10) is visible on fig. 6 (solid line).

6. MEASURED VALUES OF TEST SAMPLES

Samples mentioned on fig. 5 were used for measuring. Samples were 35 cm long, width of conductive paths was 1 mm and distance between conductive paths was 4 mm. Strip was ended with SMD resistor 51 Ω and the sample was connected to the network analyzer ANRITSU MS4622B by BNC connector. Because only one output of the analyzed is used, we figure out only S_{11} parameter, which is sufficient for comparison with theoretic results. The measurement was done in frequency band from 0,5 to 2 GHz. Measured values of S_{11} parameter are on fig. 6 (intermittent line).

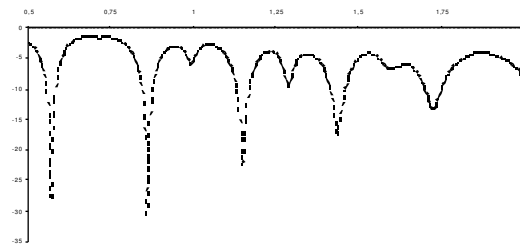


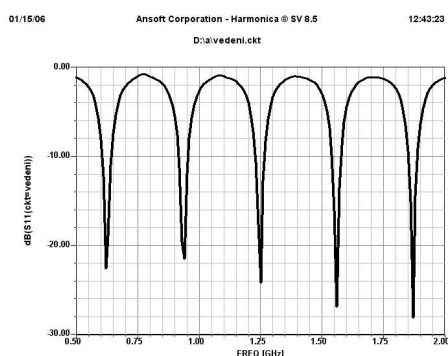
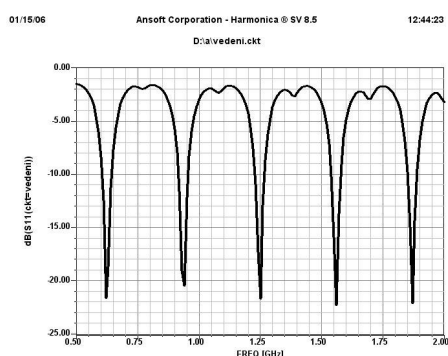
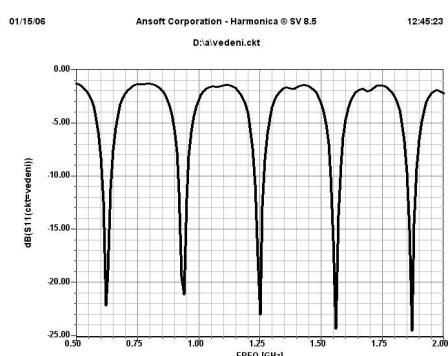
Fig. 6.: Calculated and measured S_{11} parameter

7. CONCLUSIONS

The model of coplanar strip, which is RLC network, can be used only for frequency range where the strip behaves as system with concentrated parameters. The measured and theoretical results are the evidence. At higher frequencies the individual resonant frequency are same, but the scope of characteristics is considerably different. Question is what makes such difference? The strip was simulated with software ANSOFT SERENADE SV 8.5. This software allows to microstrip simulate, so results for coplanar strip are only informative. The biggest error is to omit the tin overplating in the model build up of coplanar strip. The result of simulation is shown on figures 7, 8 and 9 for strips created:

- by copper thickness 70 μm
- by tin thickness 70 μm
- by combination of copper thickness 50 μm and tin thickness 20 μm

The influence of tin in technological progress production of printed circuit boards is evident here.

Fig. 7. Copper thickness 70 μm Fig. 8. Tin thickness 70 μm Fig. 9. Copper thickness 50 μm and tin thickness 20 μm

The real cross-sectional shape of the conductive path has another effect on resulting characteristic thereby correct functionality. Where misted of supposed an ideal rectangle a trapezoidal form or an „ arc" form (fig. 10.) have to be included in the model. These shapes depend on used resistive cover, which serves as mask for etching.



Fig.10. Cross-sections of conductive ways

The created mathematical model has to be improved by modeling of electrolytic tin overplating and real cross-section of the path. Primary parameters calculation looks as more difficult problem, than it looks at the beginning. Designed calculation of S11 parameter is correct and can be used in new mathematical model.

Acknowledgement

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REFERENCES

- [1] HOFFMAN, K.: *Planární mikrovlnné obvody*. ČVUT Praha, 2000. ISBN 80-01-02115-7
- [2] MAYER, D.: *Teorie elektromagnetického pole*. ZČU Plzeň, 1996. ISBN 80-7082-201-5
- [3] ŽALUD, V.: *Moderní radioelektronika*. BEN-Technická literatura, Praha 2000. ISBN 80-86056-47-3
- [4] PROCHÁZKA, T., BARTOŇ, Z.: *S-parametry*. www.elektrorevue.cz, 2002
- [5] Hewlett Packard Application Note 95-1, <http://www.hp.com/go/tmapnotes>
- [6] MACH, P., SKOČIL, V., URBÁNEK, J.: *Montáž v elektronice*. ČVUT Praha, 2001. ISBN 80-01-02392-3
- [7] BLECHA, T., HEINDL, K.: *Dvou vodičové koplanární vedení na vysokých frekvencích*. ZČU Plzeň 2004. Str. 25-28. ISBN 80-7043-300-0
- [8] BLECHA, T., HEINDL, K.: *Měření S-parametrů koplanárního vedení*. ZČU Plzeň, 2004. Str. 5-8. ISBN 80-7043-300-0
- [9] BLECHA, T., BRAUN, J., HEINDL, K., JELÍNEK, F.: *Modelling of substrate properties DISEE*, 2004, Bratislava. Str. 183-186. ISBN 80-227-2110-7
- [10] BLECHA, T., BRAUN, J., HEINDL, K., JELÍNEK, F.: *Testing and simulation of long wiring on printed circuit board*. Electronic Devices and Systems, 2004, Brno. Str. 373-376. ISBN 80-214-2701-9
- [11] HEINDL, K.: *Vliv provedení koplanárního vedení na jeho rozptylové parametry*. Písemná práce ke státní doktorské zkoušce, 2004, Plzeň.